SHAPE OF TURBULENT JET AXIS IN AN UNBOUNDED HORIZONTAL CROSS FLOW

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The problem is examined of determining the axis of a jet on the curvature of the jet, when the material of the jet and that of the surrounding medium have different densities.

We have used the same system to solve the problem as that in [1], making the appropriate assumptions. Only the most important practical case of efflux of a circular jet perpendicular to a transverse stream (Fig. 1) is considered.

The forces acting on an element of the jet (Fig. 1a) are as follows (neglecting viscous friction forces):

a) the aerodynamic pressure force of the cross flow, which is assumed to be proportional to the component of cross flow velocity head normal to the element, i.e.,

$$dP = C_n \frac{\rho_w (w \sin\alpha)^2}{2} b\,dl \tag{1}$$

(we consider the curvature of the jet in the plane of symmetry to be small compared with the thickness of the jet), and

b) the gravitational force, directed vertically,

$$dR = g\Delta\rho F dl. \tag{2}$$

The above forces are counterbalanced, according to d'Alembert's principle, by the centrifugal force on the element

$$dC = \frac{\rho_v v^2}{r} F dl. \tag{3}$$

The equilibrium condition in projection on the normal to the jet axis is

$$dC = -dP + dR\cos\alpha \tag{4}$$

or, substituting from (1)–(3),

$$\rho_v v^2 F = \frac{C_n}{2}\rho_w w^2 br\sin^2\alpha + g\Delta\rho Fr\cos\alpha. \tag{5}$$

We shall pursue the solution further, using the following assumptions:

1. The static pressure in the jet is equal to the static pressure in the undisturbed stream.

2. The component of jet momentum in a direction perpendicular to that of the cross flow, varies only under the influence of gravitational force

$$\rho_v v^2 F\sin\alpha = \rho_{v'} v'^2 F_0 + \int_0^l g\Delta\rho F\, dl. \tag{6}$$

3. The excess heat content of the jet remains unchanged:

$$c_{pv}\rho_v F v\Delta T = c_{pv'}\rho_{v'} F_0 v'\Delta T_0. \tag{7}$$

If the physical properties of the substance of the jet and those of the surrounding medium do not differ substantially, then, using the equation of state of a perfect gas with p = const, and neglecting the small variation of specific heat with temperature ($c_{pv} \simeq c_{pv'}$), we may obtain instead of (7) the condition for the jet to maintain constant buoyancy:

$$\Delta\rho Fv = \Delta\rho_0 F_0 v'. \tag{8}$$

The last solution is applicable also to the case of an isothermal jet, but with an initial density differing from that of the surrounding medium. In this case (8) should be taken as an initial assumption.

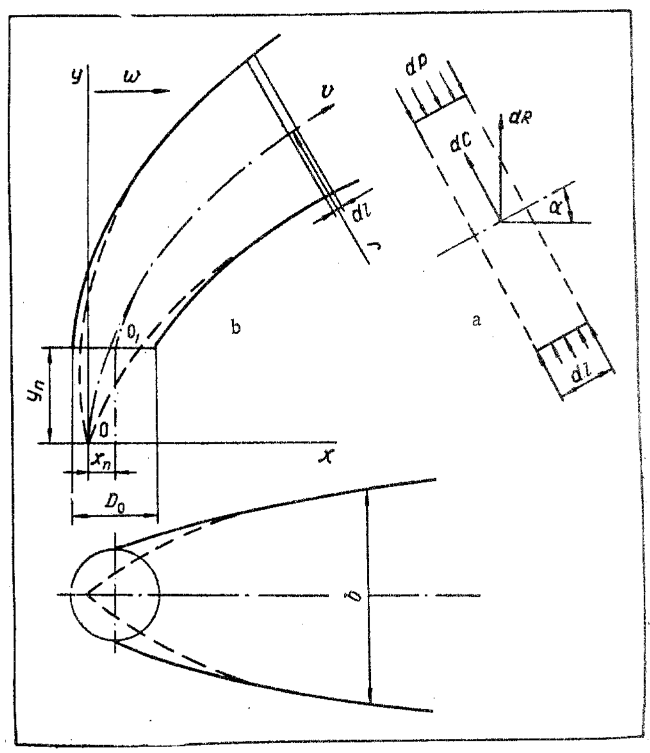

Fig. 1. Diagram of jet in the transverse stream.

4. The projection of the mean jet velocity in the direction of the cross flow is approximately equal to the velocity of the cross flow

$$v_x = w. \tag{9}$$

Experimental data [2] confirm the validity of this assumption even at a small distance horizontally from the source.

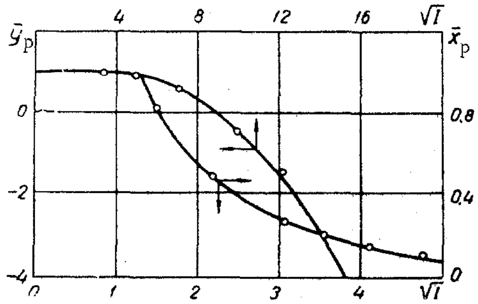

Fig. 2. Dependence of relative coordinates of the pole of the main section of a jet on the relative initial momentum.

Bearing in mind, further, that

$$r = (1 + y'^2)^{1.5}/y'', \tag{10}$$

$$\sin\alpha = y'/(1 + y'^2)^{0.5}, \tag{11}$$

$$\cos\alpha = (1 + y'^2)^{-0.5}, \tag{12}$$

and also

$$dl/v = dx/v_x, \tag{13}$$

and using (8) and (9), Eq. (5) may be put in the form

$$\rho_v v'^2 F_0 + g\Delta\rho_0 F_0 \frac{v'}{w} x = -\frac{C_n}{2}\rho_w w^2 b \frac{y'^2}{y''} + g\Delta\rho_0 F_0 \frac{v'}{w}\frac{y'}{y''}, \tag{14}$$

or in terms of the dimensionless parameters

$$\frac{J}{\bar{y}'^2} - (I + J\bar{x})\frac{\bar{y}''}{\bar{y}'^3} = \frac{2}{\pi} C_n b, \tag{15}$$

where the dimensionless groups

$$I = \frac{\rho_v v'^2}{\rho_w w^2} = \frac{T_w v'^2}{T_v w^2},\quad J = \frac{g D_0 v' \Delta\rho_0}{w^3 \rho_w} = \frac{g D_0 v' \Delta T_0}{w^3 T_w}$$

reflect, respectively, the relative influence of the initial momentum and the gravitational force on the development of the jet.

In order to solve Eq. (15), it is necessary to define the law of variation of width of the jet (Fig. 1). One of the basic factors determining mixing of the jet is the turbulence created by self motion of the jet relative to the surrounding medium. In most cases we may neglect the influence of other factors, mainly the turbulence of the cross flow at a short distance from the source, where the relative velocity of the jet is still fairly great. Then the jet width must be proportional to the trajectory of the relative motion of the jet in the surrounding medium. If we examine a jet in a cross flow conventionally consisting of three sections—an initial section, characterized by a constant-velocity core, a transition section, and a main section, where we may assume $v_x = \omega_1$—then in the main section, the relative motion of the jet will occur only in the vertical direction, and therefore $\bar{b} \sim \bar{y}$ here. Since v_x quickly becomes equal to ω, the extent of the initial and transition sections is comparatively small, and they are not examined in our analysis. From the above, we may assume the jet expansion in the main section to follow a linear law of the form

$$\bar{b} = c\bar{y}. \tag{16}$$

We shall locate the origin of coordinates, therefore, at the arbitrary point 0 (Fig. 1b), which we shall call the pole of the main section of the jet. The pole 0 does not coincide with the center of the exit section of the nozzle, 0_1, but is located at a distance from it described by coordinates $\bar{x}_p$ and $\bar{y}_p$. We shall take the initial momentum and excess heat content of the jet

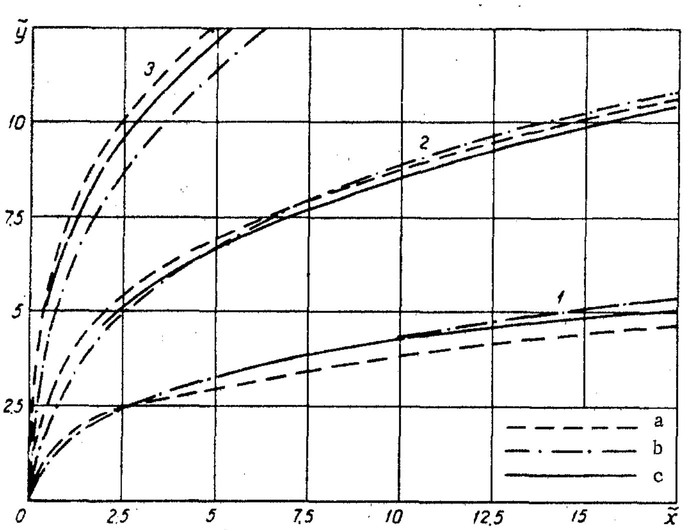

Fig. 3. Calculations of the shape of the jet axis: a) According to [3]; b) according to [4]; c) according to (19); 1) I = 4; 2) 25; 3) 100.

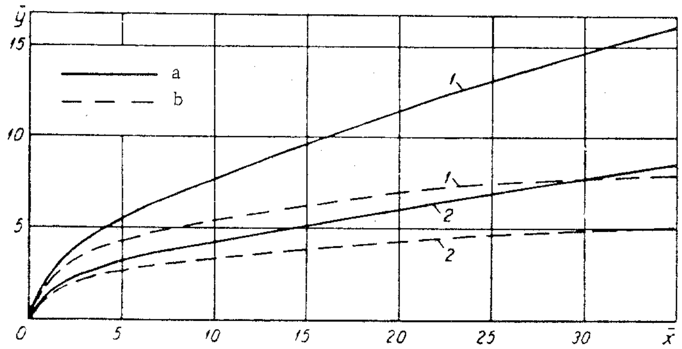

Fig. 4. The influence of buoyancy force on the shape of the jet axis (D_0 = 7 m; v' = 12 m/sec; T_ω = 283°; ΔT_0 = = 100°): 1) ω = 5 m/sec; 2) 10 m/sec; a) allowing for buoyancy force by (18); b) without allowing for it (19).

at the pole to be the same as at the exit section of the nozzle. Relation (16) is confirmed by experiments with isolated cylindrical parts, issuing from a narrow slot-shaped source of great length, the nature of whose mixing is to a large extent similar to that of the separate sections of a curved jet [3]. This relation is also supported by photographs obtained in tests with jets in a cross stream [2, 4].

Solution of the differential equation (15) with initial conditions:
when

$$\bar x = 0,\ \bar y = 0 \quad \bar y' \to \infty \tag{17}$$

gives the equation for the curvature of the jet axis in the form

$$\bar y = k\sqrt[3]{I\bar x + 1/2\, J\bar x^2}, \tag{18}$$

where

$$k = \sqrt[3]{3\pi/C_n C} = 2.11/\sqrt[3]{C_n C}.$$

If the influence of gravitational force is negligibly small, i.e., J = 0, Eq. (18) may be simplified:

$$\bar y = k\sqrt[3]{I\bar x}. \tag{19}$$

Note that it is easy to obtain the solution, in the same way, for an arbitrary initial angle of discharge, α_0, this being, for a circular jet

$$\frac{1}{k^3}\bar y^3 + I\cos\alpha_0\,\bar y = I\sin\alpha_0\,\bar x + 1/2\, J\bar x^2, \tag{20}$$

and for a plane jet

$$\frac{C_n}{4}\bar y^2 + I\cos\alpha_0\,\bar y = I\sin\alpha_0\,\bar x + 1/2\, J\bar x^2, \tag{21}$$

where D_0 has been replaced by the width of the jet δ_0 at the nozzle exit.

In order to use the solution obtained for practical calculations, it is necessary to know the numerical value of coefficient k and the coordinates of the pole $\bar x_p$ and $\bar y_p$.

It would be natural to expect that the greater the relative value of initial momentum of the jet, as denoted by graph I, the later does the main section of the jet begin. Curves showing the dependence of $\bar y_p$ and $\bar x_p$ on I are given in Fig. 2. They were obtained by comparing calculations according to (19) with experimental curves [2, 5]. It may be seen from Fig. 2 that for values of $I^{1/2}$ up to 5, $\bar y_p$ remains approximately constant and equal to 1. At the same time, the value of $\bar x_p$, which for large values of $I^{1/2}$ is close to zero, increases for small values of this parameter according to a hyperbolic law, which may be represented by

$$\bar x_p = \frac{1.6}{\sqrt{I}} - 0.26. \tag{22}$$

This horizontal shift of the pole may be explained by the fact that for small relative values of the initial momentum, under the action of the strong pressure field created by the cross flow, the jet is already deflected at the mouth of the nozzle, and issues into the stream at an actual initial angle less than the angle at which the nozzle is set.

The question of the influence of the relative value of gravitational force (group J) on the position of the pole needs experimental investigation.

From the same experimental data [2, 5], coefficient k proves to be a constant, equal approximately to 1.5.

Figure 3 shows a comparison of the results of calculations according to (19) and according to formulas obtained experimentally [2, 5], in a system of relative coordinates $\tilde x \tilde y$ with their origin at the center of the exit section of the nozzle. It may be seen there that (19) can replace, with sufficiently high accuracy, the two other formulas obtained for the different ranges of variation of group I. Calculation according to (19) also gives good agreement with the experimental data of other authors [6, 7].

Figure 4 shows curves drawn for the values of the basic parameters typical of the chimneys of electric power stations.

NOTATION

v, ρ_v, T_v) mean velocity, density, and absolute temperature at an arbitrary section of the jet, respectively; v', $\rho_{v'}$, $T_{v'}$) velocity, density, and absolute temperature at the initial section of the jet, respectively; ω, ρ_ω, T_ω) velocity, density, and absolute temperature of the cross flow, respectively; $\Delta\rho = \rho_\omega - \rho_v$, $\Delta T = T_v - T_\omega$) excess density and absolute temperature at an arbitrary section of the jet, respectively; $\Delta\rho_0 = \rho_\omega - \rho_{v'}$, $\Delta T_0 = T_{v'} - T_\omega$) excess density and absolute temperature at the initial section of the jet, respectively; F) area of an arbitrary cross section of the jet; F_0) area of the initial cross section of the jet; D_0) initial diameter of the jet; x, y) current coordinates; l) distance along the axis of the jet; b) width of the jet; r) local radius of curvature of the jet axis; α) local angle of inclination of the jet axis to the horizontal; g) acceleration due to gravity; $\bar{x} = x/D_0$, $\bar{y} = y/D_0$) relative coordinates; $\bar{b} = b/D_0$) relative width of the jet; $\bar{x}_p$, $\bar{y}_p$) relative coordinates of the pole of the main section of the jet; c_{pv}, $c_{pv'}$) specific heat of the jet at constant pressure at an arbitrary and at the initial section of the jet, respectively; C_n) resistance coefficient for flow past the jet; C) coefficient of lateral expansion of the jet.

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